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THE DIDACTIC UNIT TO PROMOTE THE DEVELOPMENT OF METRIC SPATIAL THINKING IN SIXTH-GRADE STUDENTS

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Abstract
Currently, mathematics education faces several challenges, among which low performance in spatial metric thinking stands out. This problem is reflected in students' difficulty in understanding and applying concepts related to space and measurement, which is essential in multiple disciplines and everyday situations. To address this deficiency, a pedagogical practice has been implemented with the objective of "Developing a didactic unit based on problem posing and solving to promote spatial metric thinking." This initiative is based on theories and research that supports a methodology centered on action research, proposing a problem-oriented learning approach. The designed didactic unit includes progressive activities that are presented virtually through remote learning guides. At the conclusion of the unit, an exit test was administered to evaluate students' progress. In addition, a satisfaction survey was conducted to measure students' perception of the methodology employed. The results show that the implemented methodology has generated satisfactory progress in the development of students' spatial metric thinking, demonstrating the effectiveness of teaching based on problem-solving and active learning.

Keywords: Problem solving; Spatiometric thinking; Mathematics; Intervention.

1. Introduction

Currently, mathematics education faces several challenges, among which is low performance in spatial metric thinking. This problem is reflected in students' difficulty in understanding and applying concepts related to space and measurement, which is essential in multiple disciplines and everyday situations (Clements & Sarama, 2011). Metric spatial thinking is essential for the development of exploration, description, and mastery of the environment. Geometric systems are constructed through active exploration and modeling of space, both for objects at rest and those in motion. The cognitive process progresses from the intuition of a space, given by the manipulation of objects, location in the environment, measurement, and displacement of bodies, to the conceptualization of an abstract space where geometric properties can be inferred (MEN, Curricular Guidelines, 1998).

In her study on the development of metric thinking in eighth-grade students, López (2013) from the Universidad Nacional de Colombia highlighted the importance of the measurement system in daily life. Her research focused on the strengthening of metric thinking and reinforcement of the measurement system in eighth-grade students of the Pío XI Educational Institution in Aránzazu, Caldas. The final product was a Didactic Unit with three guides based on problem situations that allowed students to experience the applicability of mathematics in their daily lives, generating greater interest and motivation for learning.

On the other hand, Estrada (2019) proposes a methodological strategy to teach spatial thinking through problem solving. His research was conducted on 13 sixth-grade students from a Colombian educational institution. Through pre-test questionnaires and the use of ICT resources, skills, interests, and difficulties in collaborative work were identified. It was observed that although there is a willingness to work in groups, students prefer to use concrete materials. In addition, difficulties were evident in activities that require spatial thinking skills, such as the calculation of perimeters, areas, volumes, and angles, which suggests the need for a more rigorous approach on the part of teachers and the educational community in general.

The importance of spatial metric thinking in the mathematics curriculum is undeniable owing to its applicability in many aspects of daily life. The study of measurement provides an opportunity to learn and apply operations, geometric ideas, statistical concepts, and notions of function. These connections complement the relationships between measurement, social studies, science, art, physical education, and other areas that require measurement or unit conversion.

Metric spatial thinking is essential for the exploration and understanding of the environment as well as for modeling objects both at rest and in motion. This cognitive process progresses from spatial intuition to abstract conceptualization, allowing the inference of geometric properties (Montaño, 2018) and underscoring the importance of the development of spatial thinking, metrics, and measurement systems in educational training.

Geometry, based on the study of geometric bodies, finds its foundations in nature and is reflected in various human expressions, such as architecture and the arts (Godino & Ruíz, 2002). The curricular guidelines of the Ministry of National Education (MEN, 1998) emphasize the understanding of measurable attributes and the development of a sense of measurement, promoting metric thinking from an early age.

Measurement estimation involves an approximate valuation of a given quantity, which is influenced by various object characteristics (Castillo-Mateo, Segovia, & Molina, 2017). Two types of magnitudes to estimate are distinguished: continuous and discrete, and they are pertinent in this work to focus on the estimation of continuous magnitudes.

Problem solving has been considered a key approach in the teaching of mathematics in schools worldwide, since it is recognized as a complex practice and an ideal framework for the construction of meaningful learning. According to George Pólya (1989), learning based on problem solving translates into meaningful competencies for students, since it involves their environment, poses challenges, and enhances skills and abilities essential for their professional and social performance, such as decision-making and research and communication skills. Pólya's theory proposes a method for problem solving that guides students' work through questions guided by the teacher, allowing problematic situations to be solved effectively.

To address this deficiency, a pedagogical practice has been implemented whose objective is to "Develop a didactic unit based on problem posing and problem solving to promote metric spatial thinking." This initiative is based on theories and research that support a methodology focused on action research, proposing a problem-solving oriented learning approach (Schoenfeld, 2016). Action research allows teachers to adapt their methods based on students' responses and needs, promoting more effective and personalized learning (Kemmis, McTaggart, & Nixon, 2014).

The didactic unit designed includes progressive activities that are presented virtually through remote learning guides. This approach not only adapts to contemporary educational needs, marked by the growing importance of technology in teaching, but also facilitates interactive and accessible learning. At the conclusion of the unit, an exit test was administered to assess students' progress. In addition, a satisfaction survey is applied to measure the students' perception of the methodology employed.

The results show that the implemented methodology has generated satisfactory progress in the development of students' spatial metric thinking, demonstrating the effectiveness of teaching based on problem-solving and active learning (Boaler, 2016). This underscores the importance of continuing to develop and refine pedagogical strategies that integrate problem-solving and the use of emerging technologies to improve performance in critical areas of mathematics education.

2. Methodology

The design of this work is based on Action Research, as proposed by Elliott 2003) and Latorre (2005), who proposed three cyclical phases of action and reflection, where the starting point is a hypothesis, the exploration of the problem, design of the proposal and its implementation, evaluation, and improvement actions. Following this model, a teaching unit was implemented for 28 sixth-grade students from a Colombian Educational Institution.

To develop the project, the design and implementation of a teaching unit based on remote learning guides is proposed. This unit contains a series of problem-solving activities to specifically promote the development of metric spatial thinking in secondary school students. In the sixth grade, each activity contained the expected learning, which was developed in three major moments. At each moment, learning develops from the formulation of elementary, introductory, and challenging problems that motivate students to carry out the proposed tasks, always considering the aspects of mathematical thinking that are intended to be promoted.

To understand the current state of student learning in relation to metric spatial thinking and to make an effective design of the teaching unit, an analysis of student performance was conducted by applying questions taken from national standardized tests. The results of this evaluation revealed several opportunities and challenges that must be addressed to improve students' academic performance.

Limited Spatial Understanding: Most students showed significant difficulties in visualizing and manipulating geometric figures in space. Many were unable to identify and relate the properties of three-dimensional objects to their two-dimensional representations.

Measurement Deficiencies: Students had problems in understanding and applying basic measurement concepts, including estimating length, area, and volume. This deficiency is reflected in frequent errors when solving problems involving unit conversion and measurement calculations.

Low Problem-Solving Performance: The ability to address and solve spatial and metric problems effectively is limited. Students tended to avoid complex problems and relied on trial and error methods rather than applying systematic strategies.

Additionally, negative attitudes towards mathematics were observed: many students expressed negative attitudes towards mathematics, including a lack of confidence in their own abilities and low motivation to participate in activities related to spatial metric thinking.

These initial results underscore the need for pedagogical intervention focused on improving spatial metric thinking and problem-solving skills. The proposed teaching unit was designed to address these deficiencies by incorporating progressive activities that develop spatial visualization, understanding of measurement, and problem-solving strategies.

Furthermore, the importance of changing students' attitudes towards mathematics through more interactive and motivating teaching approaches was identified. The implementation of remote learning guides and the use of educational technology seeks to engage students more effectively, helping them build a more positive attitude and greater confidence in their mathematical abilities.

The structure of the remote learning guide is as follows.

Table 1. Learning route

LEARNING GUIDE	EXPECTED LEARNING
The Garden	Decide on strategies to determine the relevance of the estimate and analyze the causes of errors in the measurement and estimation processes.
The journey of measurements	Estimate the measurement of length, mass, capacity, and time in the presence or absence of the objects and decide on the convenience of the instruments to use according to the needs of the situation.
The segment that wanted to be human	Estimate the measurement of angles in the presence or absence of objects and decide the convenience of the instruments to use according to the needs of the situation.
Squares	Estimate the measurement of areas, whether or not the objects are present, and decide on the convenience of the instruments to use according to the needs of the situation.
Between cubes	Estimate the volume measurement, whether or not the objects are present, and decide on the convenience of the instruments to use according to the needs of the situation.

Each tutorial contained the following elements.

Expected learning: Constitutes the learning, competence, or ability that the student is expected to develop at the end of the activity, which must be related to evidence of learning.
Exploration: At this moment, students are motivated towards new learning, recognizing their previous knowledge regarding the topic to be addressed and/or the activity to be carried out, and the importance and need for said learning.
Structuring: At this moment, the teacher performs conceptualization, explicit teaching, and modeling in relation to the learning objective. Presents the topic – does the modeling and verifies the students' understanding of learning.
Practice: Guidelines that intervene in the tasks proposed by the resources are presented so that they can generate levels of complexity for tasks that are in line with the performance levels (Minimal, Satisfactory and Advanced) and verify that the students solved the tasks correctly.
Assessment: Guidelines are presented that indicate to students the recommendations to deliver tasks that will evaluate their understanding of the knowledge constructed and the different performances of the skills developed.
Closing: At this moment the formative evaluation is carried out, the student socializes what he has learned, the activity and the feedback are evaluated
In the techniques to evaluate the information, a general rubric with evaluation scales is proposed to analyze the quantitative data at levels of inferior, minimum, satisfactory, and advanced performance with evaluation scales, where inferior corresponds to (10 -59), minimum (60–75), satisfactory (76 - 85) and advanced (86-100). Criteria based on the proposed didactic unit promote spatial metric thinking.
To evaluate the progress of the students in relation to the implementation of the proposal, a post-test was conducted in the didactic unit, a test to improve yourself with the knowledge taken from the Ministry of National Education (MEN, Superate. edu. co, 2020), which reinforces knowledge and skills regarding spatial metric thinking. This test covers 20 points to be developed, which present problem situations in the measurements of length, weight, volume, capacity, time, logical reasoning, measurement estimation, area, perimeter, angle recognition, and measuring instruments.

3. Results and discussion

The results of the implementation of the didactic unit based on problem-solving for the development of spatial metric thinking showed satisfactory progress in students. This success has been substantiated by several studies and theories in the field of mathematics education. First, the importance of a solid understanding of spatial metric thinking is well documented. According to Clements and Sarama (2011), early development of spatial skills is strongly correlated with later success in mathematics and science. This link highlights the need for effective pedagogical strategies to promote these skills at an early age. Table 2 shows how the majority of students were located at satisfactory and advanced performance levels in the general development of the didactic unit, evidencing progress in their learning.

Table 2. Performance levels in the teaching unit

PERFORMANCE LEVEL	FREQUENCY	PERCENTAGE (%)
ADVANCED	3	11
SATISFYING	10	34
MINIMUM	11	38
LOWER	5	17
TOTAL	29	100

Furthermore, the teaching unit implemented demonstrates that the proposed activities brought the students closer to the stated objective, developing thinking skills, which gave rise to the knowledge acquired, according to (Araya, 2014) “thinking skills are oriented towards understanding and improving the individual's ability to reason, and link knowledge to carry out a task or provide a solution to a problem,” in response to this reflection, the students reflected processes in which they evidence progress towards the solution of a problem and the knowledge of metric thinking space. Furthermore, the focus on problem-solving as a central methodology contributed significantly to the development of students' metric spatial thinking throughout their work with the teaching unit. George Pólya (1989) argues that problem-solving learning not only improves specific mathematical skills, but also develops transversal competences such as decision making, research and communication. These skills are crucial for students' professional and social successes.

The action research methodology used in this pedagogical practice made it possible to adapt the methodology and design of the teaching unit based on the responses and needs of the students, promoting more personalized and effective learning (Kemmis, McTaggart, & Nixon, 2014). Flexibility and adaptability are essential to address the diverse ways in which students process and understand information. As an example, the results of the implementation of the remote learning guide are shown. In the learning guide activity “Between cubes, ” 14% of the students did not identify the measurement of volumes in the presence or absence of objects. Within this percentage, there were also some students who did not answer the guide in its entirety and left most of the questions blank without answering. In turn, 34% of the students identified the measurement of volume in the presence or absence of objects. However, 52% of students estimated the measurement of volumes without considering the convenience of the instruments. None of the students showed advanced performance in the development of the guide.

Below are some of the answers proposed by the students during the development of the guide.

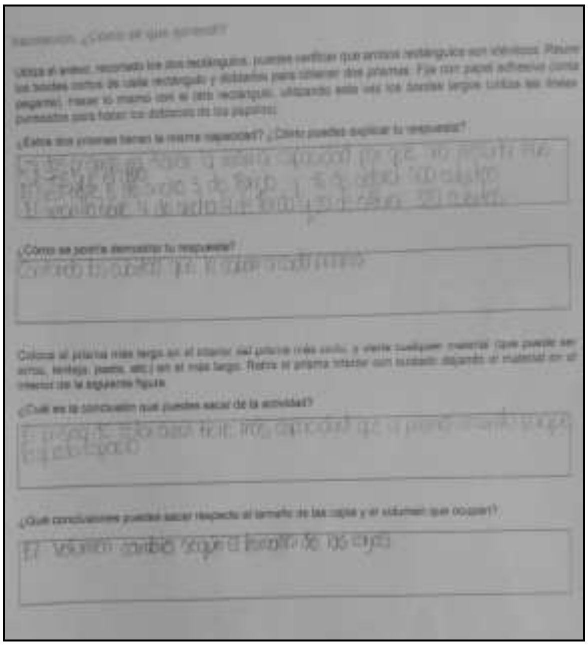

Figure 1. Response of a student to the activity at the time of assessment

For the first point in the situation presented, the student answers the first question: Do these two prisms have the same capacity? How can you explain your answer? “The two prisms do not have the same capacity because one needs more cubes than the other. The blue cubes are 5 wide, 5 deep, 16 high, and 400 cubes. The yellow one is 4 wide, 4 deep and 20 high, 320 cubes.” For the second question, how can you answer your question? The student says: “Counting the cubes that fit in each prism.” For the third question, what conclusion can you draw from the activity? The student responds: “The blue prism has more capacity than the yellow prism because it has space left.” Finally, what conclusions can you draw regarding the size of the boxes and their volume? He says: “The volume changes according to the size of the boxes.”

From the answers, it can be deduced that the student estimated the measurement of volumes without considering the convenience of the instruments. It is evident when he says ‘...because one needs more cubes than the other’, referencing how the capacities vary between a prism and the other. Later, the student proposes a relationship that clearly shows an analysis of volumes to compare the premiums when the student says “The blue one is 5 wide, 5 deep and 16 high, 400 cubes. The yellow one is 4 wide, 4 deep and 20 high, 320 cubes.” Although the relationship between the spatial dimensions of the prisms is seen correctly when determining the volumes of both prisms, the procedures that lead to the obtained results are not evident.

In the answer to the second question, the student answers “Counting the cubes that fit in each prism,” which shows the relationship in the volumes found in the previous point. The student proposed a statement but did not reinforce the answer with any procedure. For the next question, he expresses the difference in capabilities, however in the explanation, when he says “…because he has room left” it becomes simple and ambiguous because it is not explicit when differentiating which of the two cousins has room left, it is presumed that in the editorial staff did not know how to express their idea that they found from experience.

For the last point, the student correctly answers “The volume changes depending on the size of the boxes” although he did not relate the initial areas of the molds (manipulative material) with which he built the boxes. These students’ responses are a sample of their achievements in the development of metric spatial thinking through the implementation of the activities of the didactic unit.

Finally, a satisfaction survey is applied to the students in order to determine what the experience has been like during the development of the remote learning guides through which they are asked to be as sincere as possible when answering the questions. The application was applied to 29 students with five criteria: 5 (totally satisfied), 4 (satisfied), 3 (moderately satisfied), 2 (slightly satisfied), and 1 (dissatisfied). See figure 2.

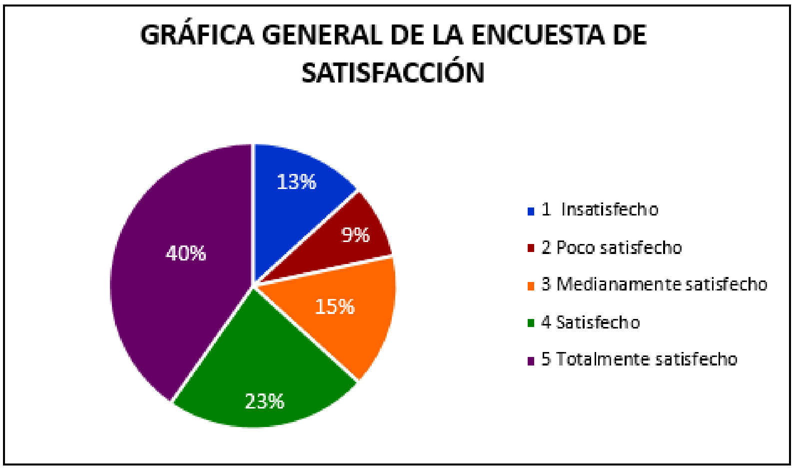

Figure 2. General graph of the satisfaction survey

The analysis of the survey shows that 40% of the respondents are completely satisfied with the participation and development in the didactic unit “The Estimation,” likewise 23% of the population expresses satisfaction, while in contrast 13% of Respondents are dissatisfied with the development and application of the teaching unit.

In open questions, in which students were asked their opinion regarding problem solving, they expressed positive comments about the importance of its application in the context for a greater understanding of the topic.

4. Conclusions and recommendations

The implementation of the didactic unit in this intervention project has shown significant progress for students in terms of the development of spatial metric thinking. The results show that the designed learning route and application of Pólya's method significantly promote the basic skills of mathematical thinking, guiding students to build solid knowledge and solve problems effectively in various contexts. The action research approach used in this project has proven to be a viable and satisfactory orientation for intervention in problematic situations. This methodology allows teachers to act in a reflective and adaptive manner, planning and implementing quality pedagogical strategies. Recognizing both difficulties and strengths during the process facilitates a more effective and contextualized intervention, resulting in a substantial improvement in student learning.

This study also confirms that Problem-Based Learning is a valuable strategy for the development of mathematical thinking in all its dimensions: spatial, metric, geometric, and numerical. This methodology not only improves the understanding and application of mathematical concepts but also fosters critical skills such as decision-making, research, and communication.

The cycle of continuous reflection inherent to action research allows students to strengthen their performance through constant monitoring and continuous evaluation. This process ensures optimal progress in relation to the stated objectives and promotes continuous improvement in teaching and learning.

The pedagogical practice has provided in-depth knowledge that strengthens professional performance. The experiences acquired have improved the curriculum and pedagogical strategies, promoting the comprehensive development of students and acquisition of essential skills for learning. In summary, the results of this study highlight the importance of integrating active and reflective methodologies in mathematics education to foster meaningful and effective learning.

At the end of the study and after a detailed analysis of the results, the following recommendations are presented with the aim of monitoring learning and strengthening student performance:

Integrating problem solving in various areas: organizing activities in different areas of the curriculum that involve problem solving. This will promote the comprehensive formation of students'skills, allowing them to apply mathematical and critical thinking skills in various contexts.

Evaluate and Contextualize the Evaluation Instruments: Carefully evaluate the evaluation instruments that will be used, placing them in the general context of the teaching unit. It is essential to clearly define assessment criteria and indicators to ensure a fair and accurate evaluation of learning.

Promote Metric Spatial Thinking: Implement activities focused on problem solving, specifically strengthening metric spatial thinking. These activities should be designed to be progressive and adaptive, adapting to the skill levels of students.

Restructure Pedagogical Practices: Redesign pedagogical practices to develop a new learning path at the institutional level. This route should be applied from the first grade of schooling to ensure a solid foundation in spatial thinking and mathematical skills from an early age.

These recommendations seek not only to maintain the progress made during the intervention project but also to expand and consolidate mathematical and critical thinking skills in students throughout their education.

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