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# MATHEMATICAL MODELING OF DISCRETE CELLULAR HIERARCHICAL SYSTEMS USING ITERATIVE NETWORKS

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**Abstract**
The methodology of structural modeling of discrete cellular-hierarchical systems is presented. A hierarchical ordered sequence of automata describing a multi-stage spatially distributed system as a set of interrelated objects (processing stages, aggregates, operations) is modeled.
Is the methodology of structural modeling of discrete cellular-hierarchical systems. Simulate the hierarchical state machines, describing an ordered sequence of spatially-distributed system of steelworks as a set of interrelated objects (processing stages, units, operations).

**Key words:** methodology, structural modeling, complex discrete systems.

Complex discrete cellular-hierarchical systems are characterized by a multi-stage process of transformation of raw materials and raw materials into finished products. The main task of structural cellular-hierarchical modeling of such systems is to construct structural diagrams of automata based on their composition. Iterative chains based on cellular interpretation are used for cell-hierarchical synthesis of complex multi-stage discrete systems. A hierarchical ordered sequence of automata describing the system as a set of interrelated objects (processing stages, aggregates, operations) is modeled. When synthesizing sufficiently complex automata corresponding to individual processing stages, theyare divided into separate cells (elementary automata) that form internal iterative chains.

**Definition 1**. An iterative chain is a composition of heterogeneous cells (automata) of varying degrees of complexity, having a sequential connection, in which the internal outputs of previous cells (automata) are the internal inputs of subsequent ones.

**Definition 2.** Discrete cellular-hierarchical system – a system of multi-level structure, which is a composition of complex automata (cells), which are divided into separate cells (elementary automata), forming internal iterative chains.
The process of structural modeling begins with the formation of functional blocks corresponding to individual processing stages, each of which is represented as a high-level state machine. Each operation corresponding to a certain stage of processing can be described as a cell without memory and is a lower-level automaton. Since operations are performed on separate technological units, each of the units, in turn, is described as a higher-level automaton (an aggregate automaton), which includes a chain of cells formed by automatic operations. This chain forms a linear iterative chain [1,2].

Similarly, at a higher level, the processing stages combine groups of technological aggregates and can be represented as a combination of cells (automata of processing stages), including chains of cells (automata of aggregates) in the form of internal iterative chains. After modeling the internal circuits of the cell hierarchy, the internal inputs and outputs of each cell at all levels are determined.

Complex industrial systems have a structure that is discrete in time, space, and alphabets. They have a complex non-linear structure of connections in terms of inputs, states, and outputs, whose alphabets are finite. Functional blocks are formed based on the processing stages, which are described by automata. When synthesizing sufficiently complex automata, they aredivided into separate cells (elementary automata) that form iterative chains or networks. Thus, in general, the production process can be represented as a composition of automata. In the resulting system, the processing stages are identified with individual automata.

Atthe next stage, the operating conditions of each automaton are formulated, i.e.the conditions of its interaction with other objects are determined, the necessary inputs and outputs of the automaton are identified, and the general law of the appearance of outputsignals depending on the effect on the inputs of the automaton is outlined. An ordered sequence of automata included in the system, which is characterized by the fact that at least one of the output nodes of each previous automaton is connected to some input node of the next automaton, forms an iterative chain.

When modeling complex automata, theyare divided into separate cells (elementary automata) that form iterative chains or networks [1-3]. Chains are iterative, because the process of determining the parameters of each cell is associated with the production process. As the semi-finished product progresses through the processing stages, the values of technological factors are recorded, which enter the machines corresponding to certain processing stages. Hence, the signals generated by the circuit automata depend on the signals of the automata (cells) generated at the previous processing stages (iterations).

For each cell, it is necessary to form its own laws of changing states and outputs, described in the form of equations or tables. From here, you can create transition and exit tables for each cell (tables 1,2).

The alphabets of inputs, states, and outputs may have different values, and their components are represented (encoded) as follows:

$k=1,...,K$ – the number of the processing stage (aggregate), $a_{kl_k j_{l_k}}$ - the component of the alphabets of inputs, where $j_{l_k}=1,...,J_{l_k}$, where $J_{l_k}$ is the value of the alphabet *of the l*-th input on *the k*-th aggregate $l_k=1,...,L_k$ – - the number of inputs (elements of raw materials, semi-finished products) on *the k*-th aggregate, $J_{l_k}$ – the value of the alphabet *of the l*-th input on *the k*-th aggregate $b_{k,m_k j_{m_k}}$ – - the component of alphabets states (technological factors) for *the k*-th aggregate, $j_{m_k}=1,...,J_{m_k}$, where $J_{m_k}$ is the value of the alphabet *of the m*-th technological factor on *the k*-th aggregate, $m_k=1,...,M_k$ and are the numbers of factors at *the k*-th stage of processing. $C_{rj_r}$ – component of the alphabets *of the r*-th output, $j_r=1,...,J_r$ - value of the alphabet *of the p*-th output, $r=1,...,R$ – output number.

So the final input alphabet is:

$$V=\{a_{111},...,a_{11J_1}\}\times...\times\{a_{1L_1 1},...,a_{1L_1 J_{L1}}\}\times...$$
$$...\times\{a_{k11},...,a_{k1J_1}\}\times...\times\{a_{kL_k 1},...,a_{kL_k J_{L_k}}\}=$$
$$=\{\sigma_{\alpha_k},\alpha_k=1,...,A_k;\ A_k=\prod_{i=1}^{L_k}J_i\}, \qquad (1)$$

where $\sigma_\alpha$ is the variant of combinations of alphabets of inputs $V[t]$, *and*$_k$ is the maximum number of combinations of alphabets of inputs on *the* $k$-th aggregate.
Final internal alphabet (the alphabet of technological factors)

$$X = \{b_{111},...,b_{11J_1}\}\times...\times\{b_{1M_1 1},...,b_{1M_1 J_{M_1}}\}\times...$$
$$...\times\{b_{k11},...,b_{k1J_1}\}\times...\times\{b_{kM_k 1},...,b_{kM_k J_{Mk}}\} =$$
$$= \{\xi_{\beta k}, \beta_k = 1,...B; B = \prod_{i=1}^{M_k} J_i\}, \qquad (2)$$

$\xi_{\beta_k}$ – a variant of the combination of alphabets on *the* $k$-th aggregate, $K$ – the number of aggregates, $m_k$ – the number of factors on *the* $k$-th aggregate.

**Table 1: Transition table for the k – th stage of processing (k-th cell of the chain)**

| $x_{k-1,1},...,x_{k-1,M_{k-1}}$ <br> $v_{k1},...,v_{kL_k}$ | $\left(\xi_{1(k-1)}\right)$ | $\dots$ | $\left(\xi_{\beta(k-1)}\right)$ | $\dots$ | $\left(\xi_{B(k-1)}\right)$ |
|---|---|---|---|---|---|
| $\sigma_{1(k)}$ <br> $\dots$ <br> $\sigma_{\alpha(k)}$ <br> $\dots$ <br> $\sigma_{A(k)}$ | $\ddots$ $\vdots$ $\cdots$ $b_{k1j_1}...b_{km_k j_{m_k}}...b_{kM_k j_{M_k}}$ $\left(\xi_{\beta k}\right)$ $\cdots$ $\vdots$ $\ddots$ | | | | |

**Table 2: Table of outputs for the k-th stage of processing (k-th cell of the chain)**

| $x_{k-1,1},...,x_{k-1,M_{k-1}}$ <br> $v_{k1},...,v_{kL_k}$ | $\left(\xi_{1(k-1)}\right)$ | $\dots$ | $\left(\xi_{\beta(k-1)}\right)$ | $\dots$ | $\left(\xi_{B(k-1)}\right)$ |
|---|---|---|---|---|---|
| $\sigma_{1(k)}$ <br> $\dots$ <br> $\sigma_{\alpha(k)}$ <br> $\dots$ <br> $\sigma_{A(k)}$ | $\ddots$ $\vdots$ $\cdots$ $c_{k1j_1},...,c_{kr_k j_{r_k}},...,c_{kR_k j_{R_k}}$ $(\tau_{\gamma(k)})$ $\cdots$ $\vdots$ $\ddots$ | | | | |

Combination of alphabets on *1*the 1st unit:

$$\xi_{\beta_1} = \{b_{111},...,b_{11J_1}\} \times ... \times \{b_{1M_1 1},...,b_{1M_1 J_1}\} =$$
$$= \{\xi_{\beta_1}, \beta_1 = 1,...,B_1; B_1 = \prod_{i=1}^{M_1} J_i\} \quad . \qquad (3)$$

Combination of alphabets on *the kth*aggregate:

$$\xi_{\beta_k} = \{b_{k11},...,b_{k1J_1}\} \times ... \times \{b_{kM_k 1},...,b_{kM_k J_{M_k}}\} =$$
$$= \{\xi_{\beta_k}, \beta_k = 1,...,B_k; B_k = \prod_{i=1}^{M_k} J_i\} \quad . \ (4)$$

Combination of alphabets on the last aggregate:

$$\xi_{\beta_K} = \{b_{K11},...,b_{K1J_1}\} \times ... \times \{b_{KM_k 1},...,b_{KM_k J_{M_k}}\} =$$
$$= \{\xi_{\beta_K}, \beta_K = 1,...,B_K; B_K = \prod_{i=1}^{M_K} J_i\}, \qquad (5)$$

$\sigma_\alpha$ - a variant of combining alphabets $V[t]$ on all aggregates:

$$\sigma_\alpha, \alpha = 1,...,A; A = \prod_{i=1}^{K} A_i \quad . \qquad (6)$$

$\xi_\beta$ - a variant of combining alphabets $x[t]$ on all aggregates:

$$\xi_\beta, \beta = 1,...,B; B = \prod_{i=1}^{K} B_i \qquad (7)$$

Final output alphabet:

$$Y = \{c_{11},...,c_{1J_1}\} \times ... \times \{c_{R1},...,c_{RJ_R}\} =$$
$$= \{\tau_\gamma, \gamma = 1,...,\Gamma; \Gamma = \prod_{i=1}^{R} J_i\} \quad , \qquad (8)$$

where $\tau_\gamma$ is a variant of the combination of input alphabets $y[t]$.

Automata for which the transition and output functionsare defined on all pairs of inputs and states are fullydefined or complete automata. Automata for whichthe input and output functions are not defined on all pairs of inputs and states are underdetermined automata. Unused pairs are not entered in the tables.

The output function looks like this:

$$\tau_{\gamma_{(k)}} = \varphi(\sigma_{\alpha_{(k)}}, \xi_{\beta(k)}) \quad . \qquad (9)$$

The combination of state alphabets at the input of the k-th cell of a chain describing a multi-stage spatially distributed system $\xi_{\beta_{k-1}}, \beta_{k-1} = 1,...,B_{k-1}; B_{k-1} = \prod_{i=1}^{M_k} J_i$, presented in the transition table, will always fall into the same block of state alphabets combination at the output of the k-th cell of the chain as a result of the transition operation $\xi_{\beta_k}, \beta_k = 1,...,B_k; B_k = \prod_{i=1}^{M_k} J_i$.

**Theorem 1.** In order for two variants of combining alphabets of states of the k-th cell of a chain describing a multi-stage spatially distributed system $(\xi_{i(k)})$ и $(\xi_{j(k)})$ to be equivalent and the iterative chain does not distinguish between them, it is necessary and sufficient that under the influence of a variant of combining alphabets of inputs $\sigma_{\alpha_k}$, the chain turns into the same combination of alphabets of outputs $\left(\tau_{\gamma_{(k)}}\right)$.

**Proof.**

(Necessity). It is necessary to prove that the variants of combinations of alphabets of states of the k-th cell of the chain are equivalent:

$$(\xi_{i(k)}) \approx_k (\xi_{j(k)}) \Rightarrow \varphi_i(\sigma_{\alpha_{(k)}}, \xi_{i(k)}) \approx_k \varphi_j(\sigma_{\alpha_{(k)}}, \xi_{j(k)}).$$

Proof to the contrary. If the output functions $\varphi_i(\sigma_{\alpha_{(k)}}, \xi_{i(k)})$ and $\varphi_j(\sigma_{\alpha_{(k)}}, \xi_{j(k)})$ are different and $\tau_{i_{(k)}} = \varphi_i(\sigma_{\alpha_{(k)}}, \xi_{i(k)})$ and $\tau_{j_{(k)}} = \varphi_j(\sigma_{\alpha_{(k)}}, \xi_{j(k)})$, then $\tau_{i_{(k)}}$ and $\tau_{j_{(k)}}$ can differ, and therefore the iterative chain will distinguish between them and $(\xi_{i(k)})$, $(\xi_{j(k)})$ are not equivalent.

(Sufficiency). You need to prove that

$$\varphi_i(\sigma_{\alpha_{(k)}}, \xi_{i(k)}) \approx_k \varphi_j(\sigma_{\alpha_{(k)}}, \xi_{j(k)}) \Rightarrow (\xi_{i(k)}) \approx_k (\xi_{j(k)}).$$

If for a variant of combinations of alphabets of inputs $\sigma_{\alpha_k}$ $\tau_{i_{(k)}} = \tau_{j_{(k)}}$, then $\varphi_i(\sigma_{\alpha_{(k)}}, \xi_{i(k)}) \approx_k \varphi_j(\sigma_{\alpha_{(k)}}, \xi_{j(k)})$. Hence, $(\xi_{i(k)}) \approx_k (\xi_{j(k)})$.

**Theorem 2.** If an iterative chain corresponding to a multi-stage spatially distributed system consists of a finite configuration of automata with a given finite set of combinations of state alphabets whose transition functions are described in tabular form, then a sequential decomposition of elementary automata can be formed for any automatonэлементарных автоматов.

**Proof.**

Since a transition table containing a finite set of possible combinations of state alphabets is given, the corresponding combinations of the k-th cell of the chain can be described as a function of certain combinations of previous cells that can be combined into one cell, which determines the sequential decomposition of elementary automata. These conclusions are confirmed by works [2-3].

A finite automaton describing a multi-stage spatially distributed system is modeled by internal chains of the cell hierarchy, in which the time argument t is not fundamental and is replaced by the spatial argument s.

Any i-level circuit automaton corresponding to a multi-stage spatially distributed system can always be modeled as an iterative i+1-level circuit, the input of which is the input of the automaton $X[s-1]_i$, and the output of which is the output of the automaton $X[s]_i$.

**Conclusion:**

A methodology for structural modeling of discrete cellular - hierarchical systems is developed. The structural elements of such systems are identified, and their main characteristics are determined. Theorems on the use of internal chains of the cell hierarchy and automaton approaches that describe the rules for modeling discrete cell-hierarchical systems are proved.

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